AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

# A-level MATHEMATICS

## Unit Mechanics 3

Wednesday 8 June 2016 Morning Time allowed: 1 hour 30 minutes

**Materials**

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

**1** At a firing range, a man holds a gun and fires a bullet horizontally. The bullet is fired with a horizontal velocity of $400\,\mathrm{m\,s^{-1}}$. The mass of the gun is $1.5\,\mathrm{kg}$ and the mass of the bullet is $30$ grams.

**(a)** Find the speed of recoil of the gun.

**[2 marks]**

**(b)** Find the magnitude of the impulse exerted by the man on the gun in bringing the gun to rest after the bullet is fired.

**[2 marks]**

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**2** A lunar mapping satellite of mass $m_1$ measured in $\mathrm{kg}$ is in an elliptic orbit around the moon, which has mass $m_2$ measured in $\mathrm{kg}$. The effective potential, $E$, of the satellite is given by

$$E = \frac{K^2}{2m_1 r^2} - \frac{Gm_1 m_2}{r}$$

where $r$ measured in metres is the distance of the satellite from the moon, $G\ \mathrm{Nm^2kg^{-2}}$ is the universal gravitational constant, and $K$ is the angular momentum of the satellite.

By using dimensional analysis, find the dimensions of:

**(a)** $E$,

**[3 marks]**

**(b)** $K$.

**[3 marks]**

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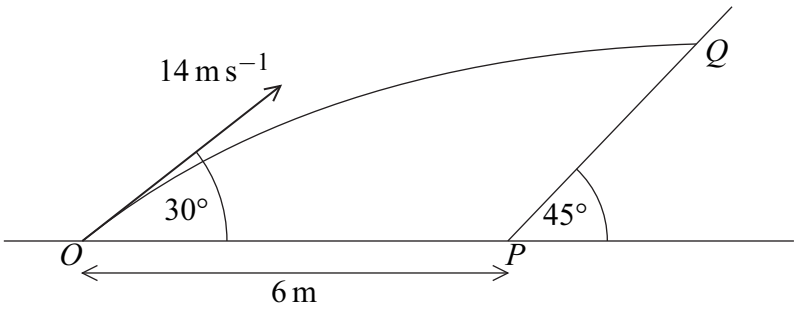
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**3** A ball is projected from a point $O$ on horizontal ground with speed $14\,\text{m s}^{-1}$ at an angle of elevation $30°$ above the horizontal. The ball travels in a vertical plane through the point $O$ and hits a point $Q$ on a plane which is inclined at $45°$ to the horizontal. The point $O$ is $6$ metres from $P$, the foot of the inclined plane, as shown in the diagram. The points $O$, $P$ and $Q$ lie in the same vertical plane. The line $PQ$ is a line of greatest slope of the inclined plane.




**(a)** During its flight, the horizontal and upward vertical distances of the ball from $O$ are $x$ metres and $y$ metres respectively.

Show that $x$ and $y$ satisfy the equation

$$y = x\frac{\sqrt{3}}{3} - \frac{x^2}{30}$$

Use $\cos 30° = \frac{\sqrt{3}}{2}$ and $\tan 30° = \frac{\sqrt{3}}{3}$.

**[5 marks]**

**(b)** Find the distance $PQ$.

**[7 marks]**

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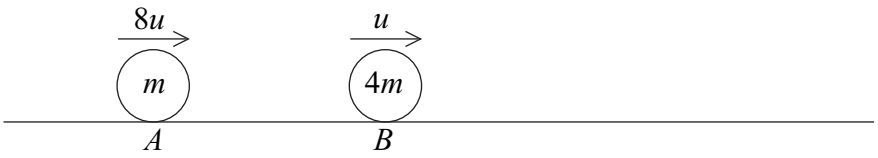
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**4** A smooth uniform sphere $A$, of mass $m$, is moving with velocity $8u$ in a straight line on a smooth horizontal table. A smooth uniform sphere $B$, of mass $4m$, has the same radius as $A$ and is moving on the table with velocity $u$.




The sphere $A$ collides directly with the sphere $B$.

The coefficient of restitution between $A$ and $B$ is $e$.

**(a) (i)** Find, in terms of $u$ and $e$, the velocities of $A$ and $B$ immediately after the collision.

**[6 marks]**

**(ii)** The direction of motion of $A$ is reversed by the collision. Show that $e > a$, where $a$ is a constant to be determined.

**[2 marks]**

**(b)** Subsequently, $B$ collides with a fixed smooth vertical wall which is at right angles to the direction of motion of $A$ and $B$. The coefficient of restitution between $B$ and the wall is $\frac{2}{3}$.

The sphere $B$ collides with $A$ again after rebounding from the wall.

Show that $e < b$, where $b$ is a constant to be determined.

**[3 marks]**

**(c)** Given that $e = \frac{4}{7}$, find, in terms of $m$ and $u$, the magnitude of the impulse exerted on $B$ by the wall.

**[3 marks]**

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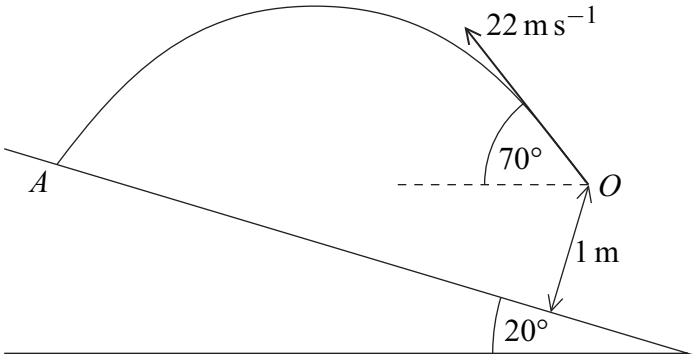
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**5** A ball is projected from a point $O$ above a smooth plane which is inclined at an angle of $20°$ to the horizontal. The point $O$ is at a perpendicular distance of $1\,\text{m}$ from the inclined plane. The ball is projected with velocity $22\,\text{m s}^{-1}$ at an angle of $70°$ above the **horizontal**. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane for the first time at a point $A$.




**(a) (i)** Find the time taken by the ball to travel from $O$ to $A$.

**[4 marks]**

**(ii)** Find the components of the velocity of the ball, parallel and perpendicular to the inclined plane, as it strikes the plane at $A$.

**[4 marks]**

**(b)** After striking $A$, the ball rebounds and strikes the plane for a second time at a point further up than $A$.

The coefficient of restitution between the ball and the inclined plane is $e$.

Show that $e < k$, where $k$ is a constant to be determined.

**[4 marks]**

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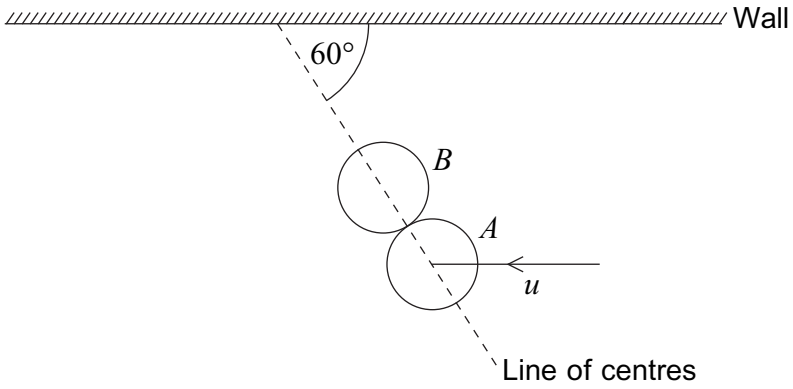
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**6** In this question use $\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$.

A smooth spherical ball, $A$, is moving with speed $u$ in a straight line on a smooth horizontal table when it hits an identical ball, $B$, which is at rest on the table. Just before the collision, the direction of motion of $A$ is parallel to a fixed smooth vertical wall. At the instant of collision, the line of centres of $A$ and $B$ makes an angle of $60°$ with the wall, as shown in the diagram.




The coefficient of restitution between $A$ and $B$ is $e$.

**(a)** Show that the speed of $B$ immediately after the collision is $\frac{1}{4}u(1+e)$ and find, in terms of $u$ and $e$, the components of the velocity of $A$, parallel and perpendicular to the line of centres, immediately after the collision.

**[7 marks]**

**(b)** Subsequently, $B$ collides with the wall. After colliding with the wall, the direction of motion of $B$ is parallel to the direction of motion of $A$ after its collision with $B$.

Show that the coefficient of restitution between $B$ and the wall is $\frac{1+e}{7-e}$.

**[7 marks]**

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**7** A quad-bike, a truck and a car are moving on a large, open, horizontal surface in a desert plain. Relative to the quad-bike, which is travelling due west at its maximum speed of $10\,\mathrm{m\,s^{-1}}$, the truck is moving on a bearing of $340°$. Relative to the car, which is travelling due east at a speed of $15\,\mathrm{m\,s^{-1}}$, the truck is moving on a bearing of $300°$.

**(a)** Show that the speed of the truck is approximately $24.7\,\mathrm{m\,s^{-1}}$ and that it is moving on a bearing of $318°$, correct to the nearest degree.

**[8 marks]**

**(b)** At the instant when the truck is at a distance of $400$ metres from the quad-bike, the bearing of the truck from the quad-bike is $060°$. The truck continues to move with the same velocity as in part **(a)**. The quad-bike continues to move at a speed of $10\,\mathrm{m\,s^{-1}}$.

Find the bearing, to the nearest degree, on which the quad-bike should travel in order to approach the truck as closely as possible.

**[5 marks]**

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## END OF QUESTIONS

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